From Recurrent Neural Network to Long Short Term Memory Architecture

Xiaoxin Wei

University De Nantes

Master Multimedia
and Data Management(MDM)

From Recurrent Neural Network to Long Short Term Memory Architecture

Application to Handwriting Recognition

Author:
Xiaoxin WEI

supervisor:
Christian VIARD-GAUDIN
Harold MOUCHERE

22nd August 2013

Abstract

Despite more than 30 years of handwriting recognition research, Recognizing the unconstrained sequence is still a challenge task. The difficulty of segmenting cursive script has led to the low recognition rate.

Hidden Markov Models (HMMs) are considered as state-of-the-art methods for performing non-constrained handwriting recognition. However, HMMs have several well-known drawbacks. One of these is that they assume the probability of each observation depends only on the current state, which makes contextual effects difficult to model. Another is that HMMs are generative, while discriminative models generally give better performance in labelling and classification tasks.

Recurrent neural networks (RNNs) do not suffer from these limitations, and would therefore seem a promising alternative to HMMs. A novel type of recurrent neural network, termed as Bidirectional Long Short-Term Memory (BLSTM) architecture, will be studied in this thesis. A sequence concatenating technique called Connecionist Temporal Classification (CTC) is applied. Finally, Three extended decoding algorithm: Levenshtein Distance(LD), full path(FD), max path(MD) are proposed insighted by HMM to have a lexicon-based classification. The system BLSTM-CTC-FP is demonstrated to be robust to lexicon-based recognition and reduce 50% error than the existing best model.

Keywords Handwriting recognition, Sequence modeling, Neural network, Long short term memory, Lexicon-based.

Contents

1 Introduction

1.1 Background

Handwriting is a skill that is personal to individuals. Fundamental characteristics of handwriting are three folds. It consists of artificial graphical marks on a surface; its purpose is to communicate something; this purpose is achieved by virtue of the mark's conventional relation to language. Writing is considered to have made possible much of culture and civilization. Handwriting recognition is the task of transforming a language represented in the spatial form of graphical marks into its symbolic representation.

Handwriting is still one of the most important communication methods even effected by the widespread acceptance of digital computers nowadays. For example, people are still requested to signed personally on documents instead of printed ones. Besides, a pen-based interfaces in digital devices are popular in the recent decades and will play a more important role in the future due to its natural interaction way for human and machine.

Therefore, Some tasks have been put forward to understand the handwriting recognition by computers [26] ,which includes *handwriting recognition, interpolation and identification. Handwriting Recognition* is the task of transforming a language represented in its spatial form of graphical marks into its symbolic representation. *Handwriting interpretation* is the task of determining the meaning of a body of handwriting, e.g, a handwritten address. Finally *Handwriting Identification* is the task of determining the author of a sample of handwriting from a set of writers, like signature verification.

On the other side, according the input data, Handwriting can be casted into online and offline data. In the online case, the two-dimensional coordinates of successive points of the writing as a function of time are stored in order. In offline case, only the completed writing is available as an 2-D image.

To some extent, *Handwriting Recognition* is the basis to the other two tasks. Although there are many existing application, the technology is not fully matured. Thus the main contribution of this thesis will focus on handwriting recognition and only the online data is considered.

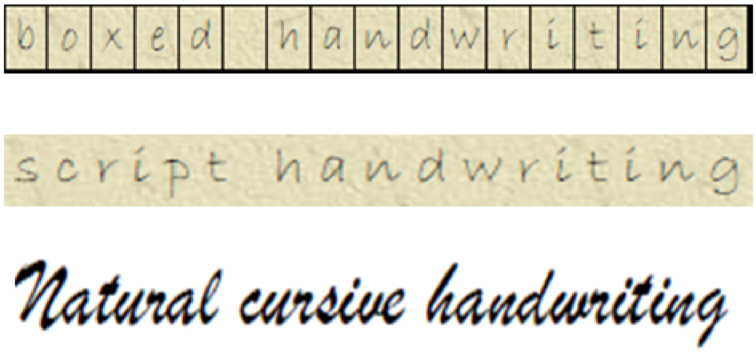


Figure 1: different kinds of handwriting sequence data input.Line 1 is box handwriting which the position of each letter is known; Line 2 is script handwriting in which each letter is isolated. Line 3 is natural handwriting which maybe cursive and unconstrained

1.2 Scope and objective

There are still two kinds of different tasks in *Handwriting Recognition*, which is to recognized isolated character and sequence. Unsurprisingly, the latter on is more difficult. Isolated character recognition has been achieved a good performance, about 99% percent for different language [34] [11] [9]

For sequence Recognition, There are different kinds of sequence input data depending on whether the position of each letter in the stream of sequence has been defined. In the first situation of Figure 1, the writer is forced to write each letter into a box, which the position is known. Also, the writer can be asked to write each letter separately. In this case, each letter will be isolated. the position is unknown but easy to obtain by applying some conditional cuts. The previous two situations can be called constrained writing because the writers writes according to some constraints. However, this does not happen in humans actual writing habits, which usually is cursive and unconstrained. Just like in the third line of the figure, each letter in a word is unknown and difficult to achieve. Therefore, it is still challenge to develop a robust and reliable system for that kind of sequence.

The main contribution of this thesis are as follows:
a). analyze the existing related sequence recognition system, point out the limitation and the relationship between BLSTM-CTC
b). apply the BLSTM-CTC system to different handwriting recognition
c). explore the optimal features for Long Short Term Memory(BLSTM)
d). extend the CTC technique to lexicon-based decoding using various

method.
e). comparison of lexicon based BLSTM-CTC with state of art models, like hybrid of HMM and NN.

1.3 Thesis Layout

The first section presents some background, the issue related to handwriting recognition and the scope, aim and contribution of this thesis. The rest of the thesis is organized as followed: Chapter 2 presents the state of the art of handwriting sequence recognition, which has been categorized into two taxonomy: implicit and explicit segment systems. Both will be listed the main technique and related papers. The purpose of this section is trying to link and distinguish the BLSTM-CTC with other systems.

In section 3, theoretical foundation of Long Short Term Memory (BLSTM). LSTM is a multi-active recurrent neural network with a more sophisticated hidden neuron called memory block. The topology of LSTM and its learning algorithm Back propagation through time (BPTT) are also referred. Finally, a forward and backward pass recurrent structure is applied to equip with bidirection.

A word level concatenating technique is discussed in section 4, called Connectionist Temporal Classification(CTC),which is used to transcribe the character-based BLSTM error onto sequence error. It is actually a HMM termilogy take advantage of some algorithm, like forward and backward, Viterbi. Thanks to HMM, different decoding methods is proposed to transplant on CTC in lexicon-based recognition.

Experiments are conducted in section 5 including framewise and temporal sequence task. Lexicon-based evaluation is used to compared with some state-of-art model.

2 State of art

2.1 Overview

The sequence recognition models can be casted into two category: explicit segment and implicit segment. The overview of these two systems can be seen in Figure 2. Both includes pre-process(here we just point out segment), feature extraction, classifier and then get the final result(segments and sequence label). There are two main distinction points between implicit and explicit, The first system needs dedicated cutting technique to be cut seen like character. By contrast, the segment of second system is based on heuristics information. Secondly.In the explicit, we can not get the result unless we know the segment, but the implicit segment system is the opposite, which it first get the result then segments.

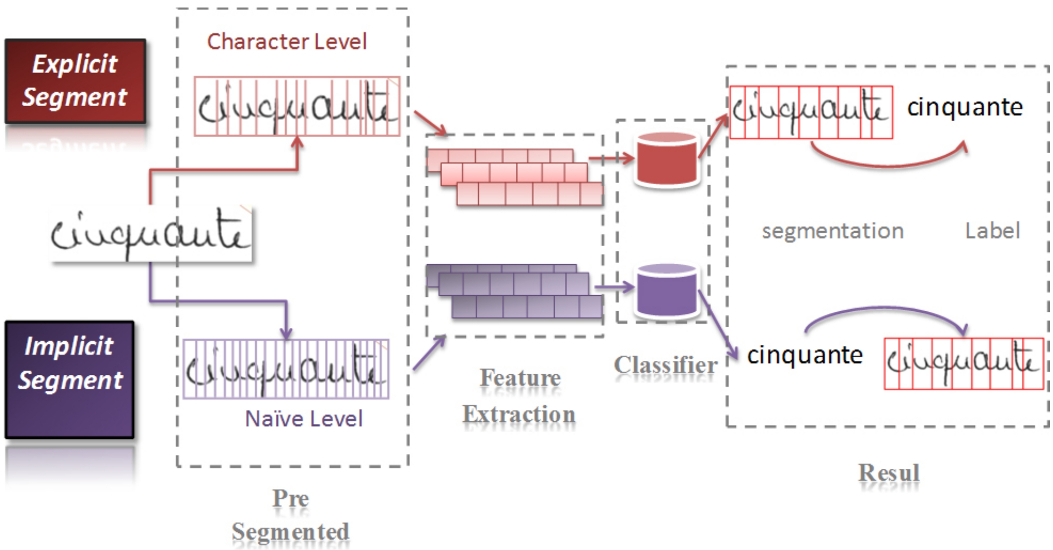


Figure 2: Sequence Recognition System Taxonomy

1.Explicit Segment

The recognition task of constrained handwriting sequence is similar to isolated character recognition as we can achieve the segmentation of the sequence. Then a sequence level can be concatenating with characters. However, to recognize the unconstrained sequence data is much more complicated since the segmentation information is unknown. One approach is to recognize individual characters and map them onto complete words, which is the same as the previous case. This needs to first classify each segment. However, segmentation is difficult for cursive or unconstrained text, at least as difficult as to recognize. In this case, if the segmentation is not reliable, the recognition will be even harder. On the other hand, segmentation is not finally defined unless the words has been recognized. This will create a circular dependency between segmentation and recognition referred to as Sayre's paradox [12].

2.Implicit Segment

Another approach to the unconstrained sequence is to simply ignore the Sayre's paradox, and carry out segmentation without "much effort",just segment them into basic strokes, slices or points, rather than characters. this will be much easier. for example, for online data, the stroke boundaries can be defined as the minimum of the velocity. For offline data, we can cut each sequence at the minimum vertical histogram. The segmentation and recognition is usually at the same. The final segmentation can not be defined unless the recognition result is known.

This Chapter has been put forward the taxonomy of sequence recognition system in Section 1. The overview of explicit segment system will be in Section 2, including the segment technique and classifiers. Section 3 will describe implicit system. Section 4 comes to the conclusion.

2.2 Explicit Segment System

For the explicit segment sequence recognition system, there are usually four steps: pre-process, segmentation,feature extraction, classifier.There are some common operations in sequence preprocessing, like noise removal, slant angle correction, smoothing, which will be the same to the implicit Segment System.

2.2.1 Segmentation

Character segment is crucial to the following step classifier in this kind of recognition system. if the segments is not correct, the result will be wrong no matter how robust the classifier is. Then researchers spend no effort to improve the performance of character segmentation. Some segmentation techniques [10] is listed as followed:

1.Dissection technique

Dissection is applied for many years, like projection analysis, connected component processing, and contextual post processing grapheme. more information of these methods is shown in the figure 3.

Space and Pitch: In machine printing, vertical whitespace often serves to separate successive characters. This property can be extended to handprinted by providing separated boxes in which to print individual symbols. The notion of detecting the vertical white space between successive characters has naturally been an important concept in dissecting images of machine print or handprinted.

Projection: projection is shown in Figure Figure 3(a), vertical projection is applied in the offline handwriting image. It is an easy matter to detect white columns between characters. but it fails to make the character O-M separation. Line 2 in (a) apply the differencing measure for column splitting and it will give a clear peak at the separated point. However, O-M case still fails. Line 3 in (b) difference after column ANDing. The image transformed by an AND of adjacent columns, which leads a better peak in between O-M.

Connect Component Analysis: Figure 3 (b) shows how connected component works. The example illustrates characters that consists of two components or more than one components will have wrong segments.

Grapheme: in Figure 3 (c), Here, a Markov model is postulated to represent splitting and merging as well as misclassification in a recognition process. The system seeks to correct such errors by minimizing an edit distance between recognition output and words in a given lexicon. An alternative approach still based on dissection is to divide the input image into sub-images that are not necessarily individual characters. The dissection is performed at stable image features that may occur within or between characters.as for example, a sharp downward indentation can occur in the center of an 'M' or at the connection of two touching characters.

2.Ligatures and Concavity

Dissection is not efficient for cursive handwriting script. Other improving features like ligatures and concavities [22] are also used for cutting point.Marks an x-coordinates as a ligatures point where the distance between y-coordinates of the upper half and lower half of the outer contour for a x-coordinate is less than or equal to the average stroke width. Besides, concavity features in the upper contour and convexities in the lower contour are used in conjunction with ligatures. The process is in following Figure 4.

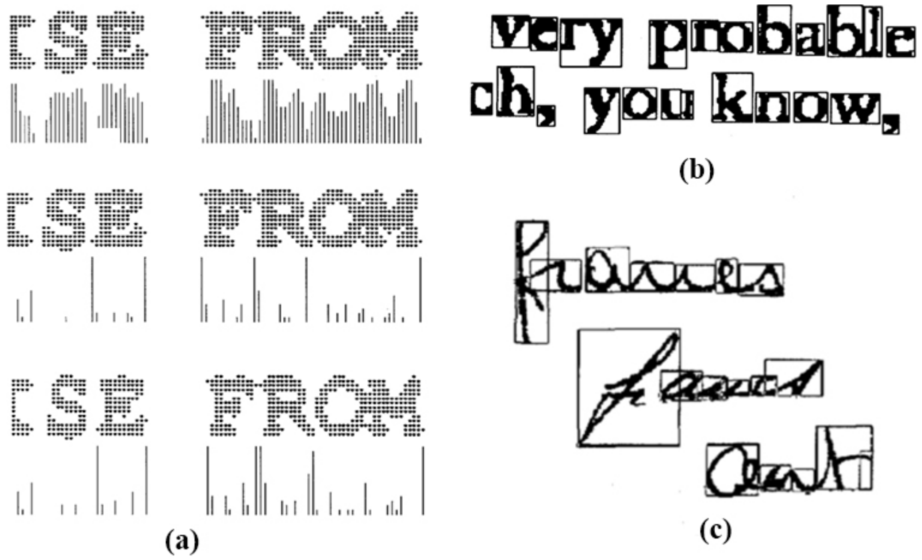


Figure 3: Dissection technique. (a)Line 1: vertical projection; Line 2: difference of vertical projection; Line 3: differencing measure of ANDing column; (b)connected component Analysis; (c)Grapheme

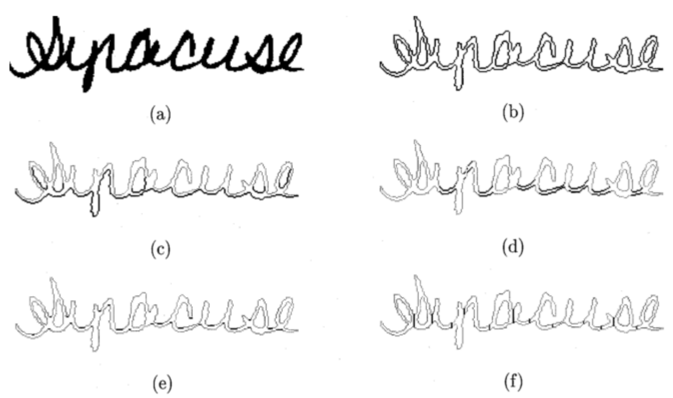


Figure 4: Segmentation of cursive script by ligature feature (a) Original test image. (b) slant normalization.(c) Splitting upper and lower contours.(c)Ligatures based on the average stroke width.(e) Concavities/convexities.(f)Segmentation points.

3.Over Segment

However, the segments result is still unreliable without considering the recognition feedback. Therefore a more modern way is a hybrid of segment and recognition. A dissection algorithm is applied to the image, but the intent is to "over-segment", i.e, to cut the sequence in sufficiently many places that the correct segmentation boundaries are included among the cuts made, as in Figure 5.

Once this is assured, the optimal segmentation is defined by a subset of the cuts made and the classification is brought to choose the most promising segmentation. The strategy is in two steps. In the first step, a set of likely cutting paths is determined, and the input image is divided into elementary components by separating along each path. In the second steps, segmentation hypotheses are generated by forming combinations of the components. All combinations meeting certain acceptability constraints(such as size, position, etc.) are produced and scored by classification confidence. An optimization algorithm is typically implemented on dynamic programming principle. One example is shown in figure 5

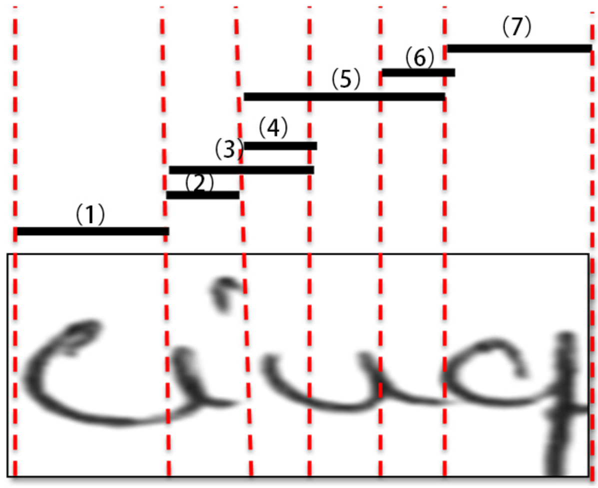


Figure 5: Over segment strategy. There are seven sub paths, and the optimized combination path is 1-2-5-7.

2.2.2 Character-base Classifier

After the segmentation, we assume the result is character-based, as the classifier and segmentation is independent, various character-based classifier can be chosen, like HMM(hidden Markov model) [25], neural network [34], support vector machine(SVM)[4].

2.3 Implicit Segment

Implicit Segment System is quite different from the previous one. It is not possible to surely say what common procedures they need. Since segmentation and recognition don't separate so clearly, and some models even don't need a segments. But we can cast them into three catalogs according to the types of the classifier in Figure 6. These are generative, discriminative, and the hybrid of both. Each of them will be discussed in this section.

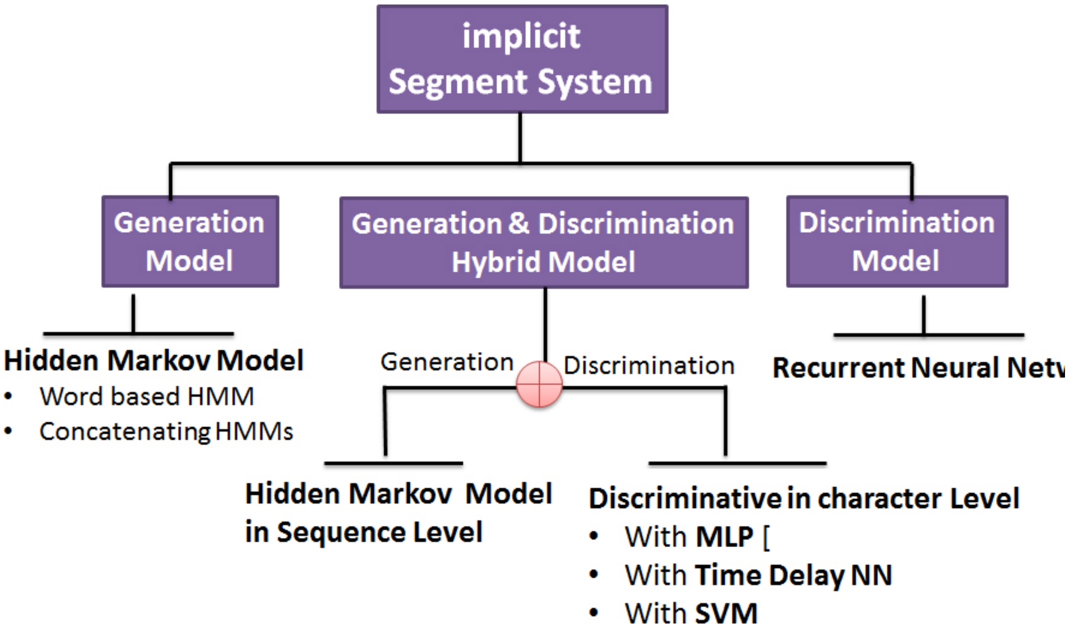


Figure 6: Implicit Segment Sequence Recognition System Taxonomy

2.3.1 Generative Model

The most famous generative model dealing with sequential data is hidden Markov model(HMM). before starting, this subsection first go through some basic theory which is also related to the technique of the thesis main word in later section, and then the model applied HMM to handwriting sequence recognition.

HMM [5] is a statistical model of Markov process, which succeeds to treat with sequential data by observations and underlying states. The Markov hypotheses define that the probabilistic description is truncated to just the current state and the predecessor state, Also, the observation is not deterministic which depend on the current state. Each HMM is represented by a set of parameter $\lambda = (A, B, \pi)$, where A denotes the transition matrix, B the matrix of emission, and π the initial state distribution. With an HMM, the first stochastic process is represented by the probability that the HMM generated observations and second by the sequence of state transitions undertaken. There are two basic types of HMM: discrete and continuous. In the case of discrete HMM, the matrix B contains discrete probabilities to each observation while in continuous, B is not directly a matrix, but represents mixtures of Gaussian or other PDFs.

There are three algorithms that are respective related to three main issues of HMM.

Forward and Backward Algorithm

One the main issue of HMM is recognition. Given several HMMs models λ_i, recognition is to classify the model with most probabilities that a given string belong tol. This is to calculate $argmax_i P(O|\lambda_i)$. The problem is to sum up all the paths of states that equal to the given observation, which is a time-consuming work. Forward and backward algorithm is generated to calculate this sum value efficiently. Since we can compare this with LSTM forward and backward algorithm in later section, we go a little further about it.

The main idea of the algorithm is to compute a forward variable $\alpha_i^t$and backward variable β_i^t, in which α_i^t denotes the probability of all the paths equal to the partial observation ending at time t with state i and meanwhile β_i^t denotes the probability of all the paths equal to given observation starting a time t with state i. Assume there are N states, T time length, then the probability of a given HMM model of an observation can be translated to.

$$P(O|\lambda) = \sum_{i=1}^{N}(\sum_{j=1}^{N}(\alpha_i^t a_i j b_j^t + 1 \beta_j^t + 1)) = \sum_{i=1}^{N}(\alpha_i^t \beta_i^t) \tag{1}$$

Forward and backward variable is like the 'save points' in the paths. It is not able to retrieve the probability of each path through this algorithm. But it doesn't matter since we only need the final sum probability, which can be achieved by adding the 'save points'.

Viterbi Algorithm

Another issue of HMM is decoding. Given a string of observation and a model λ, how to extract the optimal underlying states path. We can also compare it with the later proposed CTC decoding issue in Section It is similar to the forward and backward algorithm. We have the 'save points' of each sub path call δ_i^t. The differences to α_i^t and β_i^t is δ_i^t is not a sum value but maximum. It denotes maximum probability path ending at time t of state i among all the partial paths. Finally, the optimal state path can be obtained by traced back from time T to time *1* by choose the state with max δ_i^t at each time t. Since the it is proposed by A.Viterbi [3], the algorithm is called Viterbi .

Baum-Welch Algorithm

The training problem is crucial since a optimized model is basic guarantee to recognition and decoding. However, there is no known analytical solution that exists for the learning problem. The popular solution is iterative algorithm: Baum-Welch algorithm [2]. It is a generalized expectation-maximization(EM) algorithm for finding maximum likelihood estimates and posterior mode estimates for the parameters.

The algorithm perform alternatively between E step and M step. In E step, it calculate an expected count of the particular transition-observation pair while in M step, we update the parameters by maximizing the expected likelihood found on the E step. The parameters found on the M step are then used to begin another E step.

HMM is first distinguished itself by speech sequence processing and recognition. on-line handwriting is very similar to the problem of continuously speech recognition. Online handwriting can be viewed as a signal (x,y) over time, just like in speech. Then more and more researchers started to extend HMM to handwriting recognition. The work includes [32] [16] [7] [19] [37] [36]apply HMM in cursive handwriting recognition.

A classic application is handwriting word recognition using word-level HMM framework [7].The meaning of word-level is that each word in the dictionary forms an HMM λ_i, the recognition is performed by presenting the observations to all HMMs and select the model with greatest probability $P(O|\lambda_i)$ using forward and backward algorithm. Usually, it is a holistic method, segmentation is not necessary. All the classifier knows is the whole word, but it can't define which characters it contains. Therefore, we only can classify the exact word that occurs in the training set.The problem is that we have millions of words although we can have a small and limited alphabet(26 for English lowercase, and 65 for French). It is impossible to model all the word as an HMM and it will cost time for recognizing. Therefore this is always limited to small lexicon size.

The most common solution is to model each character in the alphabet as an HMM then concatenating them as a word. [32] developed a two-level based HMMs for online cursive handwriting recognition: letter-level and word level. The input data is sampling points, each of them first are represented by 6 features(x,y,δx,δy, $penup/pendown$, $x - max(x)$) then clustered into 64 centers. There is no segments needed and each point finally describe into 64 discrete value. 53 HMMs with 7-states left-right topology are formed to represent a single symbol including 52 character and one for white space.Since the penning of a script often differs depending on the letters written before and after, additional HMMs are used to model these contextual information.

[19] uses the similar structure for offline cursive handwriting . The sequence data first are segmented into letters or pseudo letters by ligature features. The letters or pseudo letters are described by some histogram feature, which need to be tantalized. For training, Each symbol HMM is discrete and characterized by a more delicates topology which takes the under-segments, over-segments and null value into consideration. The word model is made up by concatenation of appropriate letter models consisting of elementary HMM. The Viterbi algorithm is used in recognizing the letters in a word.

Compared with the previous discrete HMM, [37] apply a continuous density two-level continuous HMM for offline data. The segment is replaced by cutting the image into overlapping sliding windows, which are extracted pixel-based low level feature. Letter-based HMMs are generated by assuming the features in Gaussian mixture distribution.

In conclusion, there are some commons in word-level and concatenating HMMs. First, compared with the previous explicit segment system, the segment in HMMs is easier or even not necessary. Besides, the segment does not take all the risk as HMM have a hidden states. Second, both word-level and concatenating HMMs directly recognize the result word without knowing each observation belong to which letter in the word. It means that we get the result even not awaring the segments. The segments can then be retrieved by the Viterbi algorithm, which is quite different to explicit segment system.

On the other hand, the above HMM models suffer from some drawbacks. First, word-level HMM is limited to small lexicon size while in two-level HMMs, it is difficult to train the letter-base HMMs since the train datasets usually don't provided the segments. Either the system can segment sequence data into letter by hand or use another isolated character dataset. The latter lose the contextual information while the former is time consuming and not practical when the amount is large. Secondly, considering the a handwriting recognition system , where each HMM represents a letter, the HMM corresponding to a given letter only train by the samples of this letter. So an given symbol HMM neither takes the number of classes nor the similar symbol into account. Its generative capability does not maximize the distance between classes. Thirdly, both HMMs model are subjected to some assumption which may be restrict in practice. In discrete model, The features will lose information after quantizing. However, continuous model still suffers by assuming several Gaussian mixture model.

2.3.2 Hybrid of Generative and Discriminative Model

Given the drawbacks of HMM, the reason is mainly due to its arbitrary parametric assumption that given the estimation of a generative model from data. However, it is well known that for classification problems, instead of constructing a model independently for each class, a discriminative approach should be used to optimize the separation of classes.

As a consequence, by combining HMMs and discriminative model, such as neural network(NN) it is expected to take advantage of both. The researchers has explored to apply different kinds of NN associated with HMM to boost the performance of handwriting recognition. such as MLP, TDNN,RNN.

Table 1: state-of-art model:HMM Hybrid with different kinds of Neural Network applied to Handwriting recognition

#	NN	Author	Segment & Feature	Detail (function of NN, raining)
1	MLP	Rigoll, G, Kosmala A, offline	-segment free. each point is represented by 5 online and 9 bitmap	-VQ is replaced by a neural network,. -Both are trained jointly by Maximum Mutual Information(MMI) principle
2		Brakensiek, A online		
3		Marukatat Sanparith online	-segment free. each point is represented by 15 features	- emission probability are approximated with predict of MLP. -iteratively train. First letter achieved, then K-Means algorithm applied to parameter re-estimate.
4		Yong HaurTay offline	-letter hypothesescut for NN; 140 grapheme features. -discrete HMM using image frame and vector quantization	- NN as character-based classifier and approximate the mixtureGaussian for HMM. -1.NN is trained by isolate letter produced by Viterbi of an additional discrete HMM.Then it is refined several times by the hybrid resulet.
5	TDNN	Schenkel online	-segment free, each point is extracted 9 features	-NN as letter-based classifier and approximate the mixureGaussian for HMM -training for 3 steps, TDNN with character level(hand segmented),nil class train, word level train
6		S.Jager online		-TDNN for character-base -training on hand-segmented data with Viterbi constrained to the same duration in each state, then same training with unconstrained Viterbi; word level

7		Emilie Caillault online		Global discriminative training.Mixing ML,MMI and TDNN.
8	**RNN**	Joachim Schenk online	-segment free, each point is extracted same to []	-RNN for character classifier, the outputs approximates HMM density function -RNN trained by isolated character.

However, Neither the structure of how to combine NN and HMM nor how to train them is surely final defined. Thus the optimal hybrid modeling with NN and HMM is still an open question. Some related work is shown in the following Table 1

In the initial combination paper No.1 and 2 in Table 1 [28] [8], VQ in a discrete HMM is realized by replacing the usual k-means VQ by a neural network. It allows to eliminate the quantization error. Maximize the MMI principle of HMM enables to backprobagate the error to train the neural network. Paper No.3 [24] is a similar case but apply to continuous HMM, where the emission probability is approximated with predict of NN.

Although the early system contain neural network, but it is not able to involve much discriminative information. Paper No.4 in Table 1 [33] is a breakthrough in the merge of NN and HMM. NN is the character-based classifier. The isolated training data is extracted by Viterbi from baseline recognizer. The output of NN divided by the letter class prior are used directly as observation probability for the letter-level HMM. A ligature HMM and word HMMs are also designed in the system. The training process can be iterated by using the segment from the hybrid model.

The function of NN in Paper No.5 in Table 1 [29] is same to No.4[33] as approximate the Gaussian density function.

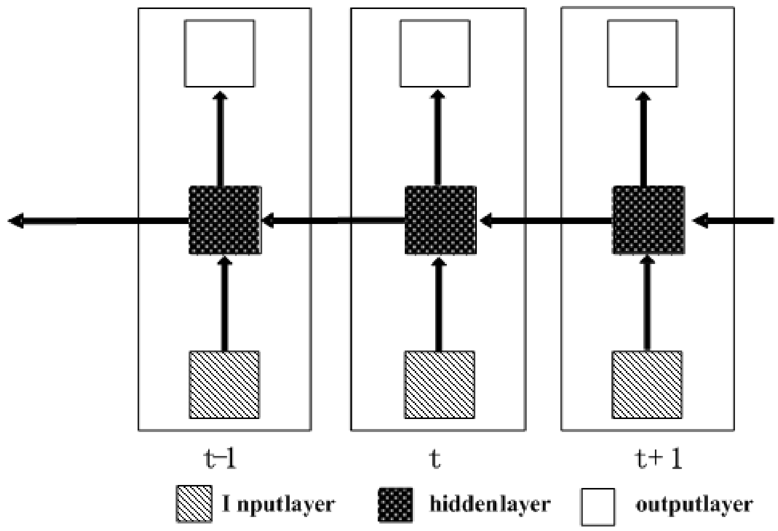
Paper No.5 [31] describe a similar structure as Paper No.4, where the mixture Gaussian is replaced by outputs of NN divided by the probability class except RNN is able to memorize and each output node of RNN is linked to just one state of a HMM. The problem is the amount of training data required for neural nets, which can be higher than for pure HMM system.

2.3.3 Discriminative Model

Although the NN-HMM hybrid well outperforms the HMMs model, it's still in the generation loop and constrained by distribution assumption. Discriminative models doesn't suffer from this problem. However, it is difficult to design a sequence recognition model only with discriminative model since it lack the ability to handle time varying sequences.

Recurrent Neural Networks are a particular category of ANNs, which is capable of storing memories and dynamically driven compared with other feedforward NN like MLP. There are various topology of RNN, one classic example is that there is one hidden layer. All the inputs are connected to hidden layer which is fully connected to itself. The recurrent link is actually a time delay connection, which if unfolded can be drawn as in Figure 7. The weights learning always use the algorithm of backpropagation through time.

Figure 7: Classic Recurrent Neural Network



But at the same time, RNN has its own problem when applied to handwriting recognition:

the current architecture fails to have long term memory. The reason is that the influence of a given input on the hidden layer, and therefore on the network output, either decays or blows up exponentially as it cycles around the network's recurrent connection. This shortcoming is called *vanishing gradient problem.* more theoretical information can be found in [18]

[6] which point out that it is hard for RNN to bridge gaps of more than 10 time steps between relevant input and target events, which is obviously unpractical for handwriting sequence.

The traditional RNN can not be directly used to sequence labeling since the objective function is unable to concatenating the sequence error.

A.Gers has solved the above problems. The gradient vanish problem is solved a multi-active structure of RNN called Long Short Term Memory (LSTM) [13] , while a HMM terminology objective function [15] is generated to deal with the second problem. Both techniques will be illustrated in the following section and will be the main work of this thesis.

2.4 Summary

This section first introduces the taxonomy of handwriting sequence recognition system, which is explicit and implicit segment. In the explicit system, the sequence recognition is obtained after the segment is decided, while we directly get the recognition result in implicit model, which can then be used to achieve the segment. Besides, the state-of-art of handwriting recognition system is then listed to further their feature and own problem. For explicit segment model, it suffers in the ambiguity and unreliability of segment process. On the other hand, implicit model is outperformed in generative classifier and the hybrid of generative and discriminative. However, the existing models are limited by the HMM generation condition.

The sequence recognition model BLSTM-CTC is similar to the previous discussed HMM-NN hybrid except it is fully discriminative. Like the hybrid, an NN structure is used for character-based discrimination, which is role of BLSTM. Besides, in the hybrid, HMM is responsible for sequence concatenating while in BLSTM-CTC, it use some forward backward algorithm and HMM idea to translate the sequence error into LSTM model. All will be detailed discussed in the later section.

But here we can first forecast the superiority compared with the previous HMM-NN hybrid models. First, the character-based discriminative classifier is trained by the data which is exactly sequence data containing contextual information without hand segment or isolation procedure. Second, character-based classifier can also fulfill the word-level recognition by a concatenating objective function. All is due to the main difference between BLSTM-CTC and the existing hybrid which get rid of the constraints of HMM.

3 Long Short Term Memory

3.1 Overview

As discussed in previous section, recurrent neural networks is able to deal with sequence data by storing variant time series, However, it fails to have long term memory.In this section, a discriminative model called Long Short Term Memory will be discussed, which is capable to deal with long time sequence. LSTM was first introduced by Sepp Hochreiter [17], The main idea of LSTM is to design a constant error cell by multi-active hidden neurons. The structure of the neurons has been modified over ten years to have better performance.

The rest context will be organized as followed: The evolution of LSTM structure is shown in Section 1, which includes input, output, forget gate and peephole connection. After defined the neurons, Section 2 describe different topology. A directional technique is added in section 3 along with the algorithm. Section 4 describe the learning algorithm needed: backprobagation through time(BPTT). Summary is given in Section 5.

3.2 LSTM architecture evolution

This subsection will be illustrated evolution of multi-active unit used in LSTM. First we define some annotation as followed:

- Subscription $w_{jj'}$ is from unit j to j'. As traditional classic RNN architecture, there are input layer, recurrent layers and output layer. The hidden layer is constructed by hidden neuron, which is called memory block(MB). One memory block can contain one or several memory cell(MC). For annotation of the nodes, i denotes the unit index in the input layer. Greek letter ρ, π, ϕ is the subscription of input gate, output gate, and forget gate in the recurrent layer. c refer to one of the cells in a block. s_c^t is the state of cell c at time t. Note that only the cell output connected to other block in the hidden layer, other LSTM activation like state is only visible in the block. We use h to refer to the cell output from other blocks in the hidden layer.

- Inputs and Outputs of Units: trigger to read and write x_j^t is the network input to any unit j at time t. For example x_i^t is the network input from unit i in the input layer at time t. y_j^t is value of x_j^t after activation. For example, any cell output unit after activation at time t in a block can be expressed as y_c^t. f is the activation function of the gates while *g,h* are activation function of cell input and output respectively.

- Units Number Let I be the number of nodes in input layer, K be the size of output layer and H be the number of cells in the hidden layer. C denotes the number of memory cells in one block.

3.2.1 Input and Output Gate

Memory cell was introduced by J.Schmidhuber [6] to have a constant error flow. The main idea is that as time goes, the NN is able to protect stored information from perturbation while outputs also require protection against perturbation. Therefore an input and output gate is generated in the memory cell, where input is the switch of reading while output is the trigger to writer. The cell in the hidden layer is not able to receive the information to store unless the input gate is open. Also, only when the output gate is open, can the cell emits the store information to output layer or to recurrent link.

Thus, Memory cell is actually a multi-active neurons with input and output gate activation. The architecture of memory cell is shown in Figure 8 The equation is shown as followed:

Input Gates :

$$x_\rho^t = \underbrace{\sum_{i=1}^{I} w_{i\rho} x_i^t}_{from\, input\, layer} + \underbrace{\sum_{h=1}^{H} w_{h\rho} y_h^{t-1}}_{from\, recurrent} \tag{2}$$

$$y_\rho^t = f(x_\rho^t) \tag{3}$$

Cells :

$$x_c^t = \sum_{i=1}^{I} w_{ic} x_i^t + \sum_{h=1}^{H} w_{hc} y_h^{t-1} \tag{4}$$

$$s_c^t = \underbrace{s_c^{t-1}}_{previous\ state} + \underbrace{y_\rho^t g(x_c^t)}_{\substack{current\, state\times \\ input\, gate}} \tag{5}$$

Outputs Gates :

$$x_\pi^t = \underbrace{\sum_{i=1}^{I} w_{i\pi} x_i^t}_{from\, input\, layer} + \underbrace{\sum_{h=1}^{H} w_{h\pi} y_h^t}_{from\, recurrent} \tag{6}$$

Cell Outputs :

$$y_c^t = y_\pi^t h(s_c^t) \tag{7}$$

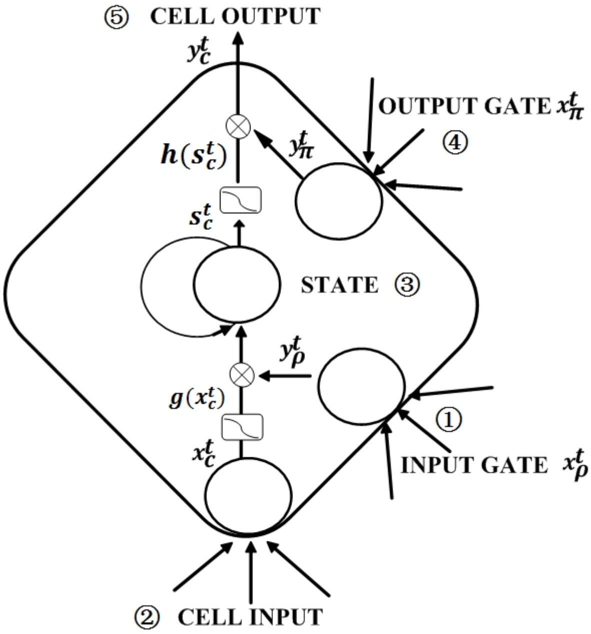


Figure 8: LSTM memory block with one cell, containing input gate and output gate. The self-recurrent(with weight 1.0) indicates feedback with a delay of one time step.There are two operation: a squarish activation and a multiplier. The number shows the updating steps.

3.2.2 Forget Gate activation: button to forget

The original LSTM allows information to be stored across arbitrary time lag, and error signals to be carried far back in time. However, if we present a continuous input stream, the cell states may grow in unbounded fashion. It is also not practical since we need to reset the cell states occasionally in the real task, e.g, at the beginning of a new character sequence.

Thus, an adaptive forget gate is added [14]. which is the same to previous LSTM except the accumulate states not adding the previous directly but with some adjustment. Architecture can be seen as Figure 9

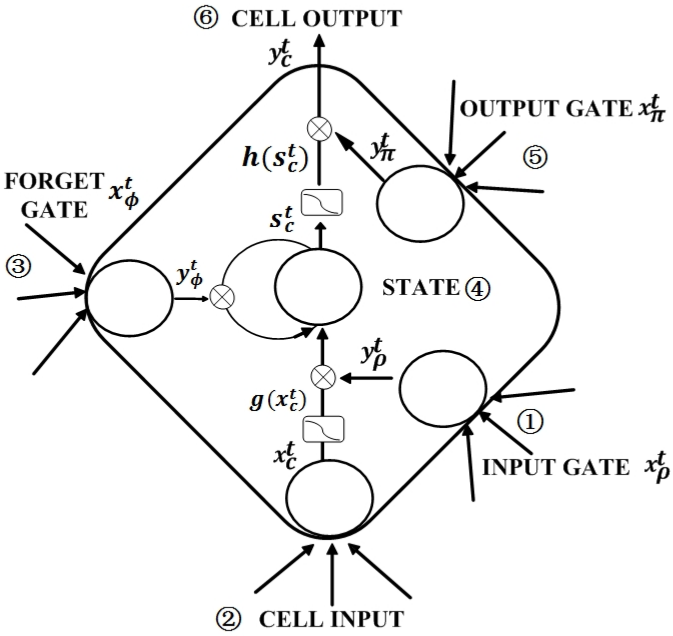


Figure 9: LSTM memory block with one cell,adding forget gate, which is an adjustment to the previous stat. The number indicates the updating order.

The forget gate is defined as same as input and output gate, and calculated before cell state.

$Forget\ Gates:$

$$x_\phi^t = \sum_{i=1}^{I} w_{i\phi} x_i^t + \sum_{h=1}^{H} w_{h\phi} y_h^{t-1} \tag{8}$$

$$y_\phi^t = f(x_\phi^t) \tag{9}$$

$Cells\ states:$

$$s_c^t = \underbrace{b_\phi^t s_c^{t-1}}_{\substack{previous\ state \\ adjustment}} + \underbrace{y_\rho^t g(x_c^t)}_{\substack{current\,state\times \\ input\,gate}} \tag{10}$$

3.2.3 Peephole Connection: immediate supervisor

LSTM is equipped with long short term memory and able to reset, but how do it know how long of memory the model needs. For example, when output gate is close, each gate receive none from the cell output. The solution is to allow all gates to inspect the current cell state even when the output gate is closed. The adding connection is called "peephole" [13], which can be founded in Figure 10

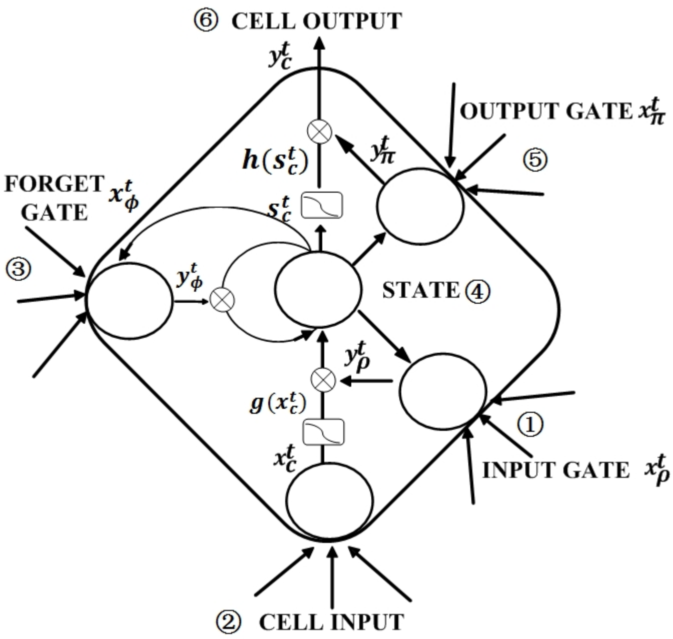


Figure 10: LSTM memory block with " peephole" connection, with which all the gates can supervise the cell state.

So state function stay the same, while each gate function modified:
Input Gates :

$$x_\rho^t = \underbrace{\sum_{i=1}^{I} w_{i\rho} x_i^t}_{from\,input\,layer} + \underbrace{\sum_{h=1}^{H} w_{h\rho} y_h^{t-1}}_{from\,recurrent} + \underbrace{\sum_{c=1}^{C} w_{c\rho} s_c^{t-1}}_{from\;cell;state} \tag{11}$$

$$y_\rho^t = f(x_\rho^t) \tag{12}$$

Forget Gate :

$$x_\phi^t = \sum_{i=1}^{I} w_{i\phi} x_i^t + \sum_{h=1}^{H} w_{h\phi} y_h^{t-1} + \sum_{c=1}^{C} w_{c\phi} s_c^{t-1} \tag{13}$$

$$y_\phi^t = f(x_\phi^t) \tag{14}$$

State function is calculated as , Noting that the input and output gate is related to previous $t-1$ state, output gate is refer to current t state:

$OutputsGate$:

$$x_\pi^t = \sum_{i=1}^{I} w_{i\pi} x_i^t + \sum_{h=1}^{H} w_{h\pi} y_h^t + \sum_{c=1}^{C} w_{c\pi} s_c^t \tag{15}$$

$$y_\pi^t = f(x_\pi^t) \tag{16}$$

3.3 Topology

After clearly discussing the architecture of memory block in LSTM, which includes an input gate, output gate and forget gate along with peephole connection. The next step is to define the whole neural network layout.

In terms to application, the more difficult the task is, more dense and complicated connection is needed. for example, we can:

1. Increase the cell number in a block.
2. Increase recurrent neurons.
3. Direct shortcut from input layer to output layer.

But more dense the network connection is, more time will be needed for training and testing.For experiments proves that one cell in a block is sufficient to handwriting recognition. Thus a not so dense topology is introduced and applied to the experiment in Chapter 5.The topology is shown in Figure 11, in which input layer is all connected to the hidden layer(gates and cell inputs). Besides, The cell outputs are fully time delay recurrent to the hidden layer and feedforward to the output layer. There is only one cell in the hidden layer constrained by the figure size. In hidden layer, more cells added will increase the input stream to cell inputs and gates.

Here supposed a classification task with output layer size O. input layer size I. We can set block size in hidden layer to H, There are C cells in one block, which share the same input, forget and output gate. The total number weights can be added by the following three items:

$From\ Input\ Layer$:

$$\underbrace{I \times H \times C}_{To\ Cell\ Input} + \underbrace{3 \times I \times H}_{To\,Gates} \tag{17}$$

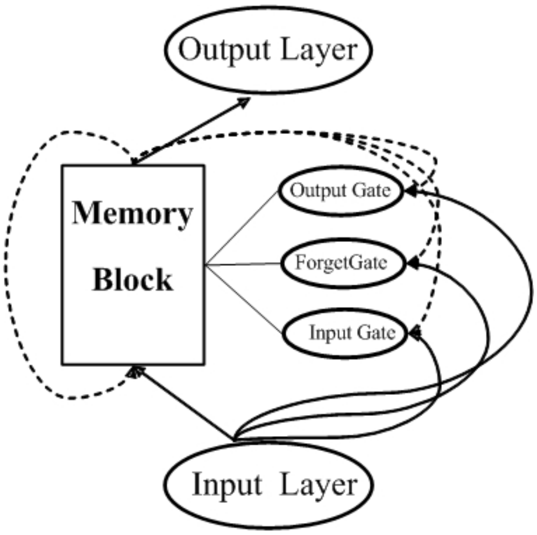


Figure 11: LSTM topology with one single-cell block in the hidden layer. The solid line indicates the feedforward connection(from input layer & to output layer), and the shade line refers to the recurrent link from cell output.noting that the peephole connection is not drawn in the figure due to the size limits.

$$From\ Output\ Layer: \quad \underbrace{O \times H \times C}_{From\ Cell\ Output} \tag{18}$$

$$From\ Recurrent: \quad \underbrace{(H \times C) \times (H \times C)}_{To\ Cell\ Output} + \underbrace{(H \times C) \times (3 \times H)}_{To\ Block\ Gates} + \underbrace{3 \times H}_{\substack{Peephole \\ Connection}} \tag{19}$$

3.4 Backprobagation through Time: learning weights

Backpropagation is the most widely used tool in the field of artificial neural networks. The Basic backpropagation has been well applied to feedforward neural network. Its further extension, called backpropagation through time (BPTT) was first introduced in [20] , which can deal with dynamic system like recurrent neural network.

Here is the BPTT algorithm pseudo-code applied to LSTM can be detailed seen in [13].

3.5 Bi-direction Extension: BLSTM

The previous LSTM with input, forget and output gate is capability with long time lag, state reset and resolve temporal distance. But all states are accumulated forward. In real task, the system is usually requested to take backward information into consideration. For example, the cursive handwriting script is one character connected with another, Thus the character after is as important as the character before.

[30] proposed a bi-directed RNN structure to overcome the limitation of single direction. The idea is to split the state neurons of a regular RNN in a part that is responsible for positive time direction(forward states) and the negative direction(backward states). Outputs from forward states is not connected to backward states. This is like two separate hidden layer which both linked from input layer and meets at output layer. The unfolded topology is shown in figure 12.

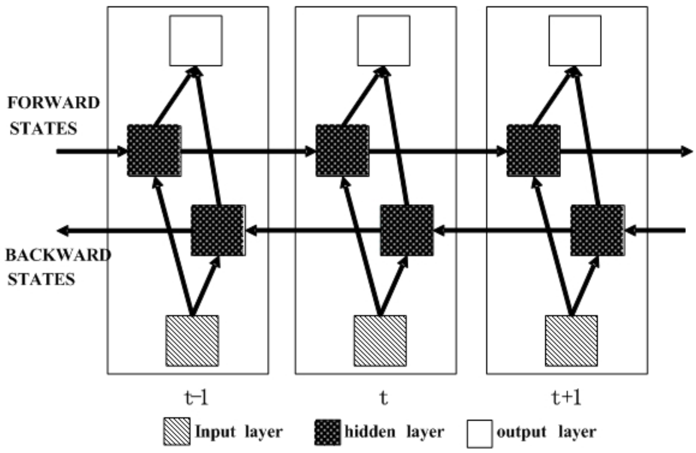


Figure 12: General structure of the bidirectional recurrent neural network shown unfolded in time for three time steps.

The bi-direction structure are also can applied to LSTM, just modifying the hidden neurons by memory blocks. The extended structure can be called BLSTM(bi-direction long short term memory).The training is the same algorithm as stated in Section 3.3 because there are no interaction between the two types of state neurons. However, BPTT is used the forward and backward pass procedure is slightly complicated because the update of state and output neurons can no longer be done one at a time. It will need to save time series in one sequence for both direction.

Supposed the one piece of sequence is from time 0 to time T, the forward and backward pass algorithm is followed by:

Forward Pass feed all input data for the sequence into BLSTM and calculate all predict output.

- Do forward pass for forward states (from time 0 to time T),save all the cell output through time
- Do forward pass for backward states (from time T to 0),save all the cell output through time
- Do forward pass for output layer by add two saving result

Backward Pass Calculate the error function derivation for the sequence used in the forward pass

- Do backward pass for output neuron
- Do backward pass for forward states (from time T to time 0). Then do backward pass for backward states (from time 0 to T)

3.6 Summary

This section first reviewed the architecture of memory blocks in LSTM. It contains an input gate, a forget gate and an output gate along with peephole connection, which is to directly link the gates with state. This structure enable the neural network with have long time dependency, state resetting and instant supervising. Then topology and the widely used learning algorithm call backpropagation through time is also discussed. Finally, considering the practical task, an enhanced BLSTM is described to have forward and backward state recurrent.

4 Connectionist Temporal Classification

4.1 Overview

In Section 2.4, we forecast the BLSTM-CTC is similar to HMM-NN model, which an NN is responsible to character discrimination and an HMM structure for sequence modeling. Thus, there are two steps first is to find an NN, BLSTM is that kind of NN illustrated in Section 3. It can not only classify letter but also store memory and indicate a letter starting and ending point in the sequence. What makes difference is the second step. Get insight from HMM, CTC can concatenates in sequence level using some HMM techniques, like forward and backward algorithm, but without constrained by the distribution assumption.

Three issue need to be considered: First, how to express from character level to sequence level. A multi-to-one mapping rule and forward backward algorithm is illustrated in Section 4.1; Second, What is the objective function? Some equations are described in Section 4.2; Finally, Section 4.3 talk about recognition problem: decoding and propose three lexicon-based decoding algorithm.

4.2 From Timestep Error to Sequence Error

In the Section 3.3, the learning is based on the t time error. Actually in temporal classification task, we don't know the t time's error instead the whole sequence error. So this section temporal classification is how to back propagate the sequence error to each t time using a HMM framework [15] on the output layer.

Let the training S be a set of training examples, each element of S is a pair of sequences(x, z). Sequence x is an input sequence given length T where $x = x_1, x_2, \ldots, x_T$ We refer to the distinct points in the input sequence as timesteps, while z is a target sequence with at most as long as the input sequence $z = z_1, z_2, \ldots, z_U, U \leq T$.

Supposed that each timestep output is conditionally independent, so the given the elements $\pi \in L^T$ is the joint probability of each output y_k^t, π is called the path ,y_k^t is the softmax normalized output of the NN at time t unite $k(k \in |L'|$, where L' is the alphabet adding a blank option, $L' = L \cap blank$.

$$p(\pi|x, S) = \prod_{t=1}^{T} y_{\pi^t}^t \tag{20}$$

Next is the definition of a many-to-one mapping $\mathcal{B} : L'^T \rightarrow L^{\leq T}$ from the set of paths onto the set L of all possible labellings. The operator $\mathcal{B}$ is executed by removing the first the repeated labels and then blank from the paths. $\mathcal{B}(a - ab) = \mathcal{B}(-aaa - -abb) = aab$ $('-'$ $means$ $blank)$. The probability to a given sequence label $z \in L^{\leq T}$ is the sum probability of all the paths π which have $\mathcal{B}(\pi) = z$.

$$p(z|x) = \sum_{\pi \in \mathcal{B}^{-1}(z)} \prod_{t=1}^{T} y^t_{\pi_t} \tag{21}$$

As the time sequence is long, there are a plenty of paths to sum, which is expensive time consuming. Fortunately, there is a similar problem in HMM to calculate the $P(O|\lambda)$. In HMM the multi paths is due to the probability relation between the hidden states and observations, While here the multi option is due to transition mapping between the path π and the target z.

Inspired by HMM, the main idea is to apply forward and backward algorithm to have saving point, just like in Section 2.3.1.

Forward variable is defined as sum probability of all paths with length t mapping to sequence label $l_{1:s/2}$. s is the index for l', where l' is sequence with the blanks added to the beginning , the end and each pair of labels in l $(|l'| = 2|l| + 1)$. Thus $s/2$ allows the sequence task to transform from blank added label to just label. $s/2$ will approximates using rounded down integer.

$$\alpha^t_s = \sum_{\substack{\pi \in N^T \\ \mathcal{B}(\pi_{1:t}) = l_{1:s/2}}} \prod_{t'=1}^{t} y^{t'}_{\pi_{t'}} \tag{22}$$

The initialization of α^t_s can be: where b means blank in the sequence.

$$\alpha^1_1 = y^1_b \tag{23}$$

$$\alpha^1_2 = y^1_{l_1} \tag{24}$$

$$\alpha^1_s = 0 \; \forall s \geq 2 \tag{25}$$

The forward algorithm is to calculate α^t based on $alpha^(t-1)$.According to the mapping rule $\mathcal{B}$, α^t_s is surely ending with label l'_s at time t. Thus the recursion can be written as :

$$\alpha^t_s = y^t_{l'_s} \begin{cases} \sum_{i=s-1}^{s} \alpha^{t-1}_i & if \; l'_s = b \; or \; l'_{s-2} = l'_s \\ \sum_{i=s-2}^{s} \alpha^{t-1}_i & otherwise \end{cases} \tag{26}$$

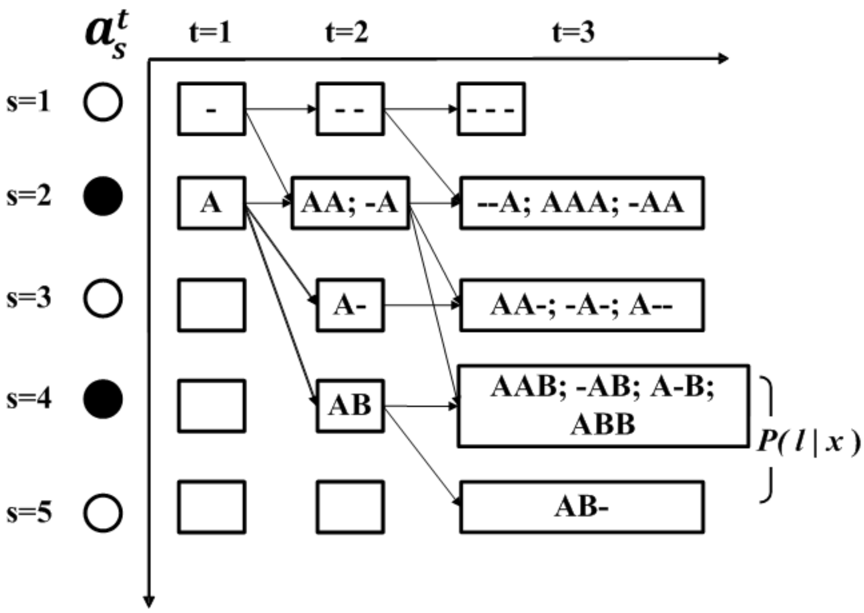


Figure 13: Forward algorithm with $l' = " - A - B - "$ $L' = 5, T = 3$, while point means blank(non-label) while black points means label. the table is filled from $t = 1\, to\, 3$, arrows denotes the recursion direction. A block means the paths that forward variable contains.

Here is the example for the recursion , we suppose $T = 3$ for observation sequence, $l = AB$, so $l' = -A - B - \;\; with\, |l'| = 5$, the algorithm of initial and recursion is shown in Figure 13

In the above figure, the paths in the block means sum probability of that forward variable. For example, when $t = 2$, there are three paths mapping to l'_3: $\pi = " - -A", "AAA", " - AA"$. Thus, $\alpha_3^2 = P(" - -A") + P("AAA") + P(" - AA")$, The forward variables can be initialized by the first column and row. Then others can be calculated by the previous column (see the arrow direction). Finally the probability of l is then the sum of the total probability of l' with and without the final blank at time T.

$$p(l|x) = \alpha_{|l'|}^T + \alpha_{|l'|-1}^T \tag{27}$$

Similarly, the backward variables is defined as the summed probability of all paths whose suffixes start at t map onto the suffix of l starting at label $s/2$

$$\beta_s^t = \sum_{\substack{\pi \in N^T \\ \mathcal{B}(\pi_{t:T}) = 1_{S/2:|l|}}} \prod_{t'=t+1}^{T} y_{\pi_{t'}}^{t'} \tag{28}$$

The initialization and recursion is

$$\beta^T_{|l'|} = y^1_b$$
$$\beta^T_{|L'|-1} = y^1_{l_1}$$
$$\beta^T_s = 0 \; \forall s \leq |l'| - 1$$

$$\beta^t_s = \begin{cases} \sum_{i=s}^{s+1} \beta^{t+1}_i y^{t+1}_{l'_i} & if\ l'_s = b\ or\ l'_{s+2} = l'_s \\ \sum_{i=s}^{s+2} \beta^{t+1}_i y^{t+1}_{l'_i} & otherwise \end{cases} \tag{29}$$

The same example as forward algorithm is given in Figure 14

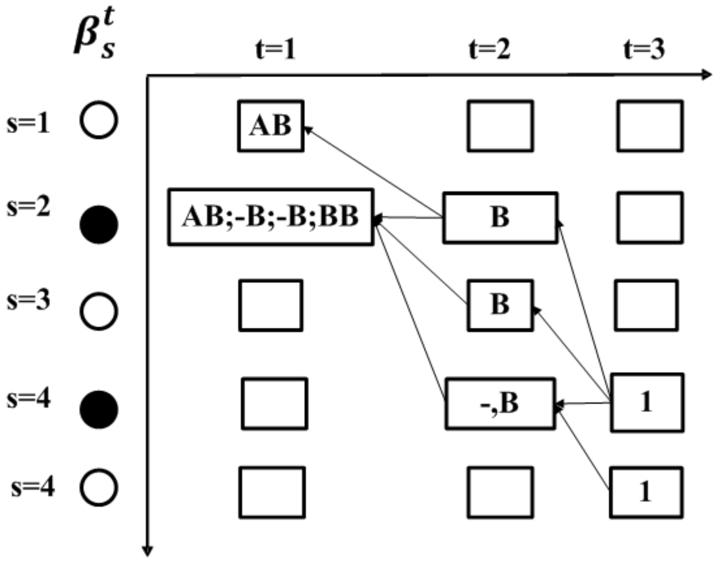


Figure 14: Backward algorithm with $l' = " - A - B - "$ $L' = 5, T = 3$, while point means blank(non-label) while black points means label. the table is filled from $t = 3\, to\, 1$, arrows denotes the recursion direction. A block means the paths that backward variable contains.

And then the label sequence is given by the sum of the products of the forward and backward variable. It means that for a labeling l, the product of the forward and backward variables at a given s and t is the probability of all the paths corresponding to l that go through the symbol l'_sat time t.

$$p(l|x) = \sum_{s=1}^{|l'|} \alpha^t_s \beta^t_s \tag{30}$$

One example is to use the above two tables(Figure 13 and 14) to have a product at each point. The result will be in Figure 15. The sum of each column through t is the the probability $p(l|x)$.

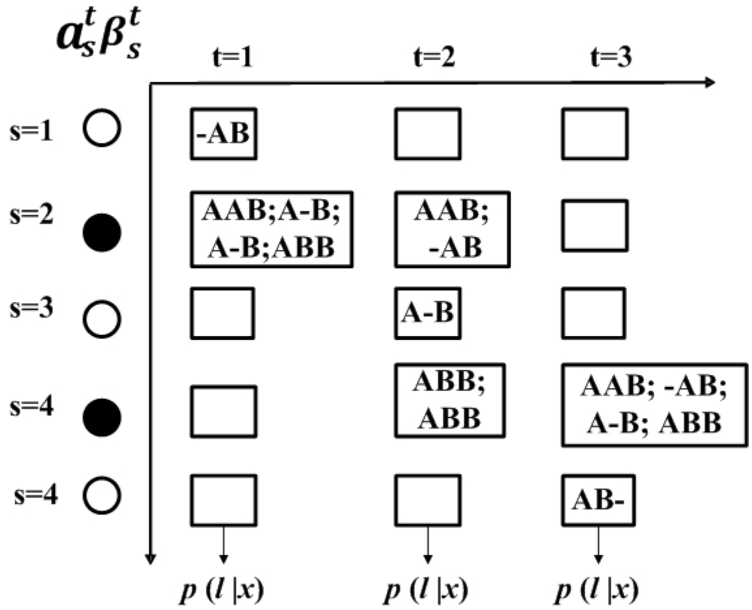


Figure 15: Product of forward and backward variable, noting that the product will be zeros if one of the variable is zeros. Sum of the column will be the probability $p(l|x)$.

4.3 Sequence Objective Function

The objective function through all the samples in dataset is defined as the negative log probability of correctly labeling the entire training set:

$$O(x, l) = -\sum_{(x,l)\in S} ln(p(l|x)) \tag{31}$$

In the training process, a coordinate gradient descend is usually used to loop from each sample. Thus in terms of a given piece of samplel,x, the deviation to the weight is equation 32, where y_k^t is the kth output of LSTM at time t.

$$\frac{\partial O(x, l)}{\partial w} = \frac{-ln(p(l|x))}{\partial y_k^t} \times \frac{\partial y_k^t}{\partial w} \tag{32}$$

The second item is solved by BPTT algorithm [20] in Section 3.4. The first item is re-expressed in 33 using forward and backward algorithm. Denoting $lab(l, k) = s : l's = k$

$$\frac{\partial p(l|x)}{\partial y_k^t} = \frac{1}{y_k^t} \sum_{s\in lab(l,k)} \alpha_s^t \beta_s^t \tag{33}$$

To backpropagate the gradient through output layer, a'^t_k is the output value before the softmax function, and

$$\frac{\partial y^t_k}{\partial a^t_k} = y^t_k \delta_{kk'} - y^t_k y'^t_k \tag{34}$$

Therefore

$$\frac{\partial O}{\partial a^t_{k'}} = \sum_{k=1}^{N} (\frac{1}{y^t_k} \sum_{s \in lab(x,k)} \alpha^t_s \beta^t_s - \frac{1}{p(l|x)}) \times (y^t_k \delta_{kk'} - y^t_k y^t_{k'}) \tag{35}$$

Noting that $\delta_{kk'}0 \quad k \neq k'$, so

$$\frac{\partial O}{\partial a^t_{k'}} = y^t_{k'} \sum_{k=1}^{N} (\frac{1}{p(l|x)} \sum_{lab(x,k)} \alpha^t_s \beta^t_s) - \frac{1}{p(l|x)} \sum_{lab(x,k')} \alpha^t_s \beta^t_s \tag{36}$$

$$= y^t_{k'} - \frac{1}{p(l|x)} \sum_{lab(x,k')} \alpha^t_s \beta^t_s \tag{37}$$

4.4 Decoding

y^t_k $\quad \mathcal{B}('--a---t') = 'at'$

'blank'	0.907	0.879	......	0.10	0.82	......	0.02
'a'	0.001	0.001	......	0.789	0.1	......	0.003
'b'	0.003	0.003	......	0.08	0.007	......	0.004
.	.	.	......	.	.	......	.
.	.	.		.	.		.
't'	0.002	0.001	......	0.001	0.001	......	0.80
'	.	.	......	.	.	......	.
'	.	.		.			.
'z'	0.001	0.0002	......	0.002	0.002	......	0.001

t=1 $\longrightarrow$ *T*

Figure 16: Decoding. The table shows the LSTM outputs at each time in the sequence. A recognition path is chosen by picking up the label with the maximum probability. Then a multi-to-one mapping rule $\mathcal{B}$ is used

Decoding is to translate plenty of paths into one sequence label l^* when we use the model to recognize sequence. The easiest way is to form a path choosing the label with the highest probability at each time step. Then using the mapping rule $\mathcal{B}$ to transcribe it as shown in Equation . The main idea can be seen in Figure 16

$$l^* = \mathcal{B}(arg\max_{\pi} p(\pi|x)) \tag{38}$$

–Lexicon-based task

The above classification task is very challenge, because even a very slight mistake will lead to the word error. And in most situation, the dictionary is known, therefore, we want to constrain the output labeling sequence according to a lexicon. Some post-progress need to be added, first method, we can try to match the above result l^* to the words W
in the lexicon. Second, we skip the process to classify the word but directly to compare the word probability given a specific word W in the lexicon $p(W|x)$. All will be detail discussed.

4.4.1 Levenshtein Distance (LD)

In this method, we directly use the result of decoding. After recognizing the label l^*, we want to compare it with the words in the dictionary. One possible approach is to calculate the edit distance, which define a way to measure the difference between two strings by transforming one string into the other, applying a series of edit operation on individual character. For every character in the first string, a sequence of the operations insert, delete and replace can be performed. The edit distance between two strings is then defined as the minimum number of edit operation needed to transform. Since scientist V.I.Levenshtein introduced that distance, it is commonly known as Levenshtein distance [23].

The edit distance D between two strings S_1 and S_2 can be calculated by a dynamic programming algorithm in recursion. And lexicon-based recognition using Levenshtein distance can be written as

$$w^* = arg\min_{word \in Dict} D(I^*, word) \tag{39}$$

The problem is that I^* is not so reliable, because it will be wrong even there is only little mistake in path π. For example, there is the path for word $'at'$ is $\pi =' ---aa---i-----t'$, the middle $'i'$ is an 'accidentally mistake' in the long sequence. To avoid the effect of this, an improved Levenshtein distance can be applied considering the operating cost to each character in the sequence. The main idea is to product the confidence of this character in I^*, like $p('i'|'--aa--i---t') = y^7_{'i'}$, then the cost to delete and substitute $'i'$ is $y^7_{'i'}$. The meaning is that more confident the result, more cost needed to pay.

Let $A(n) = b_1, b_2, \ldots, x_n$ be the classification sequence and $B(m) = b_1, b_2, \ldots, b_m$ be a given word in the lexicon. where two strings with respective length n and m,$Y_{l^*_k}$ be the confidence value of recognizing l^*_k The distance recursion value $D(r, k)$:

$$\begin{aligned} D(r,k) = min(D(r-1,k-1) + c(a_r, b_k) * Y_{l^*_r}, \\ D(r-1,k) + c(a_r, \lambda) * Y_{l^*_r}, D(r, k-1) + c(\lambda, b_k)) \end{aligned} \quad (40)$$

In Equation 40, the first item is substitution, second is deletion while the third is insertion. Only the substitution and deletion are multiplied with the confidence of the classification. Since there are several consecutive time output label equal to l^*_r, we choose $Y_{l^*_r}$ by

$$Y_{l^*_r} = \min y^t_{l^*_r}\ , where\ \pi_t = l^*_r \quad (41)$$

4.4.2 Full Path Decoding (FP)

The previous decoding method will lack the ability to reject, which we build by a post probability. As words in the dictionary is given $word|word \in Dict$, the one with the highest probability will be the winner. Therefore the problem has become

$$I^* = arg \max_{word} p(word|x) \ \ (word \ \in \ Dict) \quad (42)$$

There are plenty of path π mapping to a given $word$. Full path decoding is to consider all of these path. The problem is the same to calculate $p(l|x)$. A forward algorithm can be applied:

$$p(word|x) = \alpha^T_{|word|} + \alpha^T_{|word|-1} \quad (43)$$

The advantage is that we considered all the possibilities, which is the same criteria with training process. But it is time consuming when the dictionary size is large.

4.4.3 Max Path Decoding (MP)

Since we want a fast way to compute a given recognition, a max path probability is computed instead of the full paths added. But how to know which path is with highest probability. Fortunately, thanks to the Viterbi algorithm, we can apply it here.

Since there is no hidden states in LSTM, it is much easier. First we initialized the Viterbi operaotor δ_s^t same as forward variable α_s^t as in . Second we recursive from $t = 1$ *to* T using the following equation.

$$\delta_s^(t+1) = y_s^t \begin{cases} max(\delta_(s-1)^t,\, \delta_(s)^t) & l'_s =' b' \, or \, l'_s = l'(s-2) \\ max(\delta_(s-2)^t,\, \delta_(s-1)^t,\, \delta_(s)^t) & otherwise \end{cases} \tag{44}$$

at the same time, the trelix as the max path is recorded by:

$$\pi_t^* = \begin{cases} arg \max\limits_{i \in s-1,s} \delta_i^t & l'_s =' b' \, or \, l'_s = l'(s-2) \\ arg \max\limits_{i \in s-1,s,s-2} \delta_i^t & others \end{cases} \tag{45}$$

Since there is no hidden states in LSTM, it doesn't need to back trace like Viterbi in HMM. the probability for a given *word* is approximate to probability of its max path π as:

$$p(word|x) \approx max(\delta_{|word|}^T, \delta_{|word|-1}^T) \tag{46}$$

4.5 Summary

This Section first reviewed the Connectionist Temporal Classification technique, which can equip a recurrent neural network with the ability to sequence recognition. Modified forward and backward algorithm is used to compute the observation probability. For lexicon-based recognition task, Three kinds of decoding algorithm is proposed. First, a fast decoding method is applied using improved Levenshtein distance. Second, a full path method is calculated by the previous forward algorithm. Third, inspired by HMM, a modified Viterbi is applied to LSTM as the max path decoding.

5 Experiment

5.1 Overview

Compared with the HMM and HMM-NN hybrid, LSTM-CTC is totally a discriminative model along with the ability of long time sequence dependency without the limitation of generative distribution assumption in HMM. Three improved decoding methods are proposed for the lexicon-based task. There are various issues that need to be addressed in order to make the implementation of the system successful. We make use of an available databases IRONOFF [35] for the experiments to test the system.

There are two kinds of experiment are designed: framewise and temporal experiment. In a framewise experiment the label for each time frame is given and no sequence level technique needed. The purpose of this experiment is to compare it with other classic neural network structure, like MLP, RNN. In the second kind of experiment, The evolution of LSTM will first be considered to compared. Also we compared the model BLSTM-CTC with other state-of-art models in lexicon-base recognition to prove its superiority.

Therefore, this chapter is organized as followed: Section 5.2 describe the dataset IRONOFF. The first kind of experiment is implemented in Section 5.3 while temporal recognition is conducted in Section 5.4. Section 5.5 gives the summary.

5.2 IRONOFF Dataset

The IRONOFF database is collected by Christian Viard-Gaudin from IRCCyn laboratory in 1999. It contains both online and offline handwriting data. Different kind of data is included: isolated digits, lower case letter, upper case letter, signs and words. Both Train and Test set is previously defined. More specific number can be founded in Table 2. All the words are unconstrained and were written by about 700 different writers. The various writing style from the same word "soixante" can be seen in Figure 17.

In framewise experiment, the digit set will be used for baseline experiment. In the temporal experiment, the Cheque Word set (CHEQUE) and the whole word set(IRONOFF-197) are both tested. CHEQUE is a subset of word sequence, only with 30 lexicon. The words are from the French bank checks. For the whole word set(IRONOFF-197), there is 197 words in dictionary. Although there online and offline data, we only use the online data

Table 2: IRONOFF dataset details

Type	Training Samples	Test Samples	Total
Digit	3059	1510	4086
Lowercase	7952	3916	10685
Uppercase	7953	3926	10679
Cheque word	7956	3978	11934
English Word	1793	896	2689
French Word	19105	9552	28657
IRONOFF-197	20898	10448	31346

for experiment.

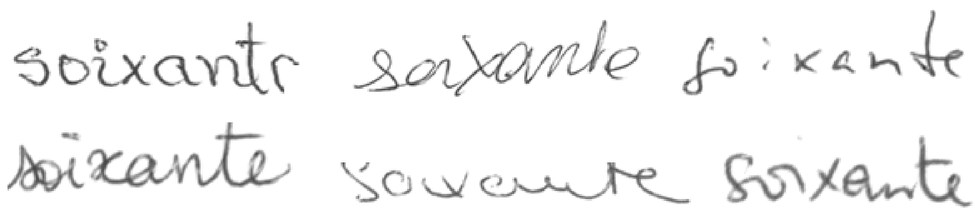


Figure 17: Word "soixante" in IRONOFF from six writers

5.3 Framewise Recognition

–Data Pre-process & Feature Extraction

The digit set online data in IRONOFF is used. Each of the samples is isolated digit comprising by invariant time length. In the pre-process, because there are two kinds of network is compared;feed-forward and recurrent. In feed forward structure, since the network need to have the fix size of one sample, each digit is first sampled by a fix number of points 30, while in recurrent network, there is no need to sample the points, because recurrent can deal with variant length . Second, size normalization is applied. All are limited to [-1,1] bounding box depending on the larger size of width and height.

7 features $(x, y, \Delta x, \Delta y, \Delta(\Delta x), \Delta(\Delta(y)), pen-on)$ are extracted to each point of the digit, which is similar in [8]

–Neural Networks topology

Table 3: Digits Results

	MLP	Classic Recurrent	LSTM	BLSTM
Frame Level	–	0.3%	0.06%	0.043%
Digit Level Last frame	2.86%	2.27%	2.06%	1.7%
Digit Level multi-to-one mapping $\mathcal{B}$	–	2.6%	2.06%	1.7%
Weights	8740	1450	4960	9910

Four kinds of NN are applied for comparison:

MLP: with one hidden layer, each input is fully connected to hidden layer, and hidden is fully connected to output.

Recurrent Neural Network(RNN): classic recurrent neural model.

LSTM: in which hidden neuron cell is a block with input, output, forget gate and peephole connection, the input layer is fully connected to neuron cell and all gates. Hidden layer is fully recurrent to itself.

BLSTM : long short term model with two independent hidden layer, both are connected to input and output layer. One is "forward state" layer, the recurrent state is based on previous information. The other is "backward state" layer, which account on the future information.

All the number of hidden neurons in the following structure is set to 30. In MLP, the input will be the whole sequence points with 210 timesteps(30 sampling points, each by 7 features), and the output size is 10. For the other three network, the input size is 7 and then it can accumulate the invariant time step length along each digit. The output size is 10 plus 1 for blank, since we assume that all the labels before the last time step is "not a digit" while the label of the last time frame is set to the digit.

–Experiment Result

The result is shown in in Table 3. In recurrent network, Both frame level and digit level recognition rate is evaluated. Frame level recognition rate is the correct frames through the total frame number while digit level the correct digit through all the digits. When concatenating to digit level, two techniques are used: mapping only depend on the the last frame or using the multi-one mapping rule $\mathcal{B}$ in Section 4.2 to transcribe the path to sequence.

$$l^* = \mathcal{B}(arg\max_{\pi} p(\pi|x)) \tag{47}$$

In this translation mapping, the output may contain not only one digit but several. The recognition rate is defined as only the sample correctly classified.

From the result, we can see that all the system achieve good result since the simplicity of the dataset. All the recurrent systems are better than MLP, where BLSTM is the best, obtaining 40% less error than MLP. Due to the large input size of MLP, its weight is nearly two times as much as LSTM. Compared the classic recurrent and two LSTM models, The interesting is that classic recurrent get different results to two evaluation of digit level, while both LSTM is the same. It means that in classic recurrent, some wrong digits will be come up not until the last frame, and LSTM and BLSTM only observe a label in the last frame. It shows LSTM structure can have more stable performance by having memory cell.

5.4 Temporal Recognition

After the base recognition to digit, BLSTM achieved the best result. Now BLSTM-CTC is applied to word sequence task using IRONOFF dataset, some experiments are organized as followed:

a. Since in word sequence, 'x' will grows from the character left to right. Thus this 'x' feature is not reasonable for character-based discriminative model. Some optional modification needed be applied for optimizing the result.

b. To further understand the cell structure in LSTM, we compare the evolution version of BLSTM in Section 3.2. First is classic RNN, then the multi-active cell with only input and output gate, then adding forget, peephole connection finally extended to BLSTM.

Lexicon-based Experiment:

c. CHEQUE and IRONOFF-197 dataset are used to compare different kinds of decoding methods. Dictionary is enlarge by adding redundant words to see the robustness and efficiency of different decoding methods.

d. The best result of c is used to compared with other state-of-art models.

–Data Pre-process & Feature Extraction

The rescaling procedure is a little different from previous step, because we need to estimate the length of the word. The procedure is as followed: first is centered at point (0,0), the height of word is in the range [-1,1], and the width is in [-number of Stroke, number of stroke], which want to adjust the width depend on the stroke number in this word.

7 features is extracted as previous. But the problem is that the x coordinate will grow as the word written, which depends on the length of the word. Since the x value is used in the feature, some normalization is needed to modify.

Lists of coordinate x normalization is:

Method a, noNorm: no normalization for x coordinates, in this case, even for the same letter, the value depend on where it occurs in the word.

Method b, winMean: a sliding window is applied. Which the normalized x equal to itself subtract the mean value of the point in this window.the windows size is set to 3.

Method c, winNorm: after Method b, then apply linear normalization divided by range in this window.

Method d, substract : each 'x' just subtract the previous 'x'.

Method e, lR: subtract the value of linear regression curve $t - x$.

Some visualization of these methods applied to the word 'seize' can be seen in Figure 18. Finally we want to define whether the previous method are contributed or just adding noise, so

Method h, wX: just remove the x-coordinate. And reduce the input to 6 dimensions.

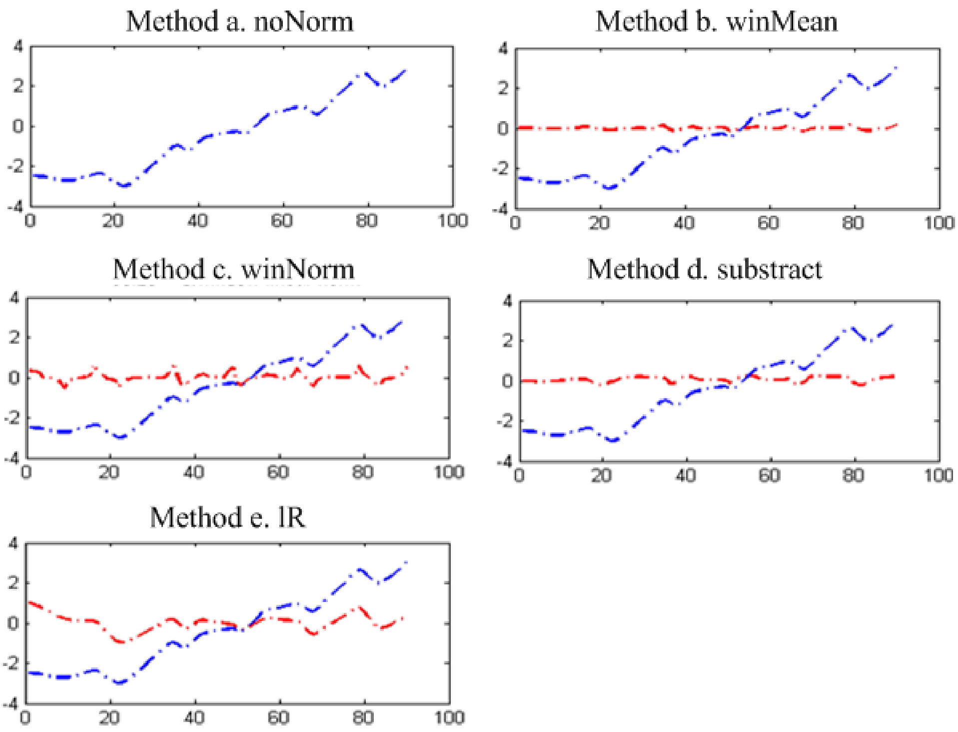


Figure 18: Different kinds of x-coordinate normalization. blue line for no normalization, which x increase as word written. red ones show different kinds of normalization, which are fluctuated between the line.

–Neural network topology

We use a BLSTM structure, which there is two independent hidden layer ('forward' and 'backward' state). Each of them is fully connected to input layer and recurrent to themselves. The Input layer size is 7. In CHEQUE set, output size is 23(22 alphabet plus a blank unit), the hidden neuron is set to 50 for each direction. Thus 25722 weights is achieved. While in full set, the output layer size is 67(66 alphabet plus blank) and the hidden neuron increase up to 80 to fit it. The hidden neuron number has not been search to optimize.

–Experiment a: Compare x- coordinate

Table 4: Normalization of coordinate 'x' in sequence on CHEQUE

	Test Samples Rate	Train Samples Rate
Method a, noNorm	67.63 %	90.2%
Method b, winMean	80.02%	95.7%
Method c, winNorm	81.55 %	99.6%
Method d, substract	80.37%	99.5%
Method e, lR	76%	96.2 %
Method f, wX	79.63 %	99.9 %

The table 4 shows that different 'x' normalization results on the CHQUE set. Evaluation method is recognition rate of the correct samples through all test set. To reduce the random phenomenon during the training, each method system has been repeated five time to have an average recognition rate.

From Figure 4, the no normalization input (blue line) is growing when word written, which is not reasonable. What we want to retrieve the trivial feature characteristic inside the letter since we can't segment the sequence to have the x coordinate for isolated letter. winMean and subtract method seems to flat after normalization without carrying too much distinction information. LR adjust full curve and may be too much depend on the sequence. The results shows that no normalization method is pretty bad in performance. LR seems over-modified. The 'winNorm' is best one. Thus, in the later experiment, we adjust the x-coordinate by method c.

–Experiment b: BLSTM evolution

BLSTM has evolved from classic RNN, multi-active cell only with input and output gate, then adding forget gate and peephole connection. Then finally extended to bidirection. IRONOFF-197 is applied to test. Evaluation method is as previous: recognition rate of the correct samples through all test set. To reduce the random phenomenon during the training, each method system has been repeated five times to have an average recognition rate. The result is shown in Table 5.

RNN is not able to deal with long time sequence. And without the reset button it is still not working. There is a sharp increase by adding forget gate to adjust the accumulated memory. Peephole connection can boost the system with about 10% increase by supervising the the inside cell state, while bidirection is also important with another 10% increase.

Table 5: IRONOFF result on evolution of LSTM

	Rec. Rate
RNN	1.62%
Input& Output Gate	7.05%
Input, Output & Forget Gate	41.43%
LSTM	50.89%
BLSTM	62.62%

–Experiment c: Lexicon based experiment: different decoding method

In the following experiment, all the result is based on the lexicon. Both CHEQUE and IRONOFF-197 are used in Table 6. Further comparison by larger lexicon size can be seen in Table 7, where IRONOFF-196 is used to add redundant words to see the effect when dictionary size extending to 300 and 500. The time is calculated seconds per sample.

We can see full path decoding achieve the best satisfying result, followed by max path and Levenshtein distance. However, as the dictionary size increase, the time to compute full path increase nearly two times. while the LD and MP nearly with the same time. All the decoding method will have a slightly drop when lexicon size enlarge, but it is still robust. It needs to be point out that [21] only achieve 87.4% when lexicon size extended to 500.

–Experiment d: CHEQUE set compared with stat-of-art model

We compared the best result in Experiment b to exsisting IRONOFF dataset, shown in Table 8. The first model [33] has been discussed in Section 2.3.2, which NN is responsible to character-based recognition and the outputs are used to approximate the observation probability. In the second model [21], two kind of feature are extracted: low level and contour based. The multi-stream character-based HMM is designed to combined two feature.

Table 6: Decoding Methods on IRONOFF-CHEQUE & IRONOFF-197

Decoding Method	Cheque word-30	Full Set-197
Levenshtein Distance(LD)	97.99%	95.16%
Full Path(FP)	99.2%	97.45%
Max Path(MP)	98.7%	96.99%

Table 7: IRONOFF-197 performance using different lexicon size

Decoding	Lexicon Size					
	197		300		500	
	acc.	time(sec.)	acc.	t	acc.	t
Levenshtein Distance(LD)	95.16%	0.033	95.14%	0.033	94.85%	0.033
Full Path(FP)	97.45%	0.065	96.64%	0.079	96.12%	0.108
Maximum Path(MP)	96.99%	0.040	96.36%	0.041	96.49%	0.042

Then the word-level HMM is constructed. The third model [1] is to combine SVM and HMM, which SVM is referred to letter classifier. and actually no HMM assumption is used in the system, but the Viterbi algorithm is used to find the best path. therefore it is an explicit system, which first get the path then the result. The final model 'bi-character' [27] aims to model the HMM which can recognized jointly with its neighboring character.

BLSTM-CTC has greatly outperformed SVM-HMM, and HMMs while it is sightly outperform NN-HMM. However our model is 50% less error than it. Besides, it should be mentioned that there are 140 sophisticated features used in this model and a refined step is applied to boosting the result. The first model without refine has 91.7% for full word set.

Table 8: Commpared with other state-of-art model

Method	IRONOFF-CHEQUE	IRONOFF-196
NN HMM hybrid [33]	98.2 %	96.1%(91.7% without refine)
Multi stream HMM [21]	90.6%	83.8%
SVM HMM hybrid [1]	76.71 %	64.53%
Bi-character model [27]	83.8%(only 500 samples)	–
BLSTM-CTC-FP	***99.2%***	***97.45%***

6 Conclusion

This thesis has applied BLSTM-CTC framework to handwriting recognition, which it is robust to sequence recognition. It is discriminative that without the assumption constraints of HMM. Besides, The model can deal with long time dependency, temporal distance supervisory and bi-direction. First we explore the inputs of the model. Then we propose three decoding methods subjected to the lexicon-based recognition task, trigger by the decoding in HMM. As the parameters of the model have not been set delicately, such as topology, number of hidden neurons, the optimization process can be fulfilled to boost the performance in the future.

References

[1] Abdul Rahim Ahmad, Christian Viard-Gaudin, and Marzuki Khalid. Lexicon-Based Word Recognition Using Support Vector Machine and Hidden Markov Model. *2009 10th International Conference on Document Analysis and Recognition*, pages 161–165, 2009.

[2] L.E.Baum and T.Petrie. Statistical inference for probabilistic functions of finite state Maarkov chains. *Analysis of Mathematical Statistics*, 37, 1966.

[3] A.Viterbi. Error bonnds for convolutional codes and an asymptotically optimum decoding algorithm. *IEEE trans.on Information Theory*, 13:260–267, 1967.

[4] Claus Bahlmann, Bernard Haasdonk, and Hans Burkhardt. Online handwriting recognition with support vector machines-a kernel approach. In *Frontiers in Handwriting Recognition, 2002. Proceedings. Eighth International Workshop on*, pages 49–54. IEEE, 2002.

[5] LE Baum, T Petrie, G Soules, and N Weiss. A maximization technique occurring in the statistical analysis of probabilistic functions of Markov chains. *The annals of mathematical statistics*, 41(1):164—-171, 1970.

[6] Yoshua Bengio, Patrice Simard, and Paolo Frasconi. Learning long-term dependencies with gradient descent is difficult. *Neural Networks, IEEE Transactions on*, 5(2):157–166, 1994.

[7] Guy Bercu, Sophie and Lorette. On-line handwritten word recognition : An approach based on hidden Markov models. In *Proc. 3rd Int. Workshop on Frontiers in Handwriting Recognition*, pages 385–390, 1993.

[8] A Brakensiek, A Kosmala, D Willett, W Wang, and G Rigoll. Performance Evaluation of a New Hybrid Modeling Technique for Handwriting Recognition Using Identical On-Line and Off-Line Data. In *Proc.5th Int.Conf.on Document Analysis and Recognition*, number 4, pages 446–449, 1999.

[9] Francesco Camastra. A SVM-based cursive character recognizer. *Pattern Recognition*, 40(12):3721–3727, December 2007.

[10] Richard G Casey and Eric Lecolinet. A survey of methods and strategies in character segmentation. *Pattern Analysis and Machine Intelligence*, 18(7):690–706, 1996.

[11] Jian-xiong Dong, A KrzyÅ¼ak, and CY Suen. An improved handwritten Chinese character recognition system using support vector machine. *Pattern Recognition Letters*, 26(:):1849–1856, 2005.

[12] Fatma El-Khaly and Maher A Sid-Ahmed. Machine recognition of optically captured machine printed arabic text. *Pattern recognition*, 23(11):1207–1214, 1990.

[13] FA Gers, NN Schraudolph, and J Schmidhuber. Learning precise timing with LSTM recurrent networks. *Journal of Machine Learning Research*, 3(1):115–143, 2003.

[14] Felix A Gers and Fred Cummins. learning to forget: continual prediction with LSTM. pages 1–19, 1999.

[15] Alex Graves and S Fernández. Connectionist temporal classification: labelling unsegmented sequence data with recurrent neural networks. In *Proceedings of the 23rd international conference on Machine learning*, pages 369–376. ACM, 2006.

[16] Unconstrained Handwritten and Word Modeling. An HMM based approach for off-line unconstrained handwritten word modeling and recognition. *iEEE Trans. on Pattern Analysis and Machine Intelligence*, 21(8):752–760, 1999.

[17] S Hochreiter and Jürgen Schmidhuber. Long short-term memory. *Neural Computation*, 9(8):1735–1780, 1997.

[18] Sepp Hochreiter, Yoshua Bengio, Paolo Frasconi, and Jürgen Schmidhuber. Gradient flow in recurrent nets: the difficulty of learning long-term dependencies, 2001.

[19] William Hu, Jianying and Brown, Michael K and Turin. HMM based online handwriting recognition. *IEEE Trans. on Pattern Analysis and Machine IntelligencePattern Analysis and Machine Intelligence*, 18(10):1039–1045, 1996.

[20] Paul J.Werbos. Backpropagation through time what it does and how to do it. *Proceedings of the IEEE*, 78(10):1550—-1560, 1990.

[21] Y KESSENTINI, T PAQUET, and AM BENHAMADOU. A multi-stream HMM-based approach for off-line multi-script handwritten word recognition. *ICFHR*, 2008.

[22] Gyeonghwan Kim and Venu Govindaraju. A lexicon driven approach to handwritten word recognition for real-time applications. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 19(4):366–379, 1997.

[23] Vladimir I Levenshtein. Binary codes capable of correcting deletions, insertions and reversals. In *Soviet physics doklady*, volume 10, page 707, 1966.

[24] Sanparith Marukatat and Thierry Artieres. Sentence recognition through hybrid neuro-Markovian modeling. In *in Proc. 6th Int. Conf.on Document Analysis and Recognition*, pages 731–735, 2001.

[25] Hiromitsu Nishimura, Makoto Kobayashi, Minoru Maruyama, and Yasuaki Nakano. Off-line character recognition using hmm by multiple directional feature extraction and voting with bagging algorithm. In *Document Analysis and Recognition, 1999. ICDAR'99. Proceedings of the Fifth International Conference on*, pages 49–52. IEEE, 1999.

[26] R. Plamondon and S.N. Srihari. Online and off-line handwriting recognition: a comprehensive survey. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 22(1):63–84, 2000.

[27] Sophea Prum, Muriel Visani, and Jean-Marc Ogier. On-Line Handwriting Word Recognition Using a Bi-character Model. *2010 20th International Conference on Pattern Recognition*, pages 2700–2703, August 2010.

[28] G Rigoll, A Kosmala, and D Willett. An investigation of context-dependent and hybrid modeling techniques for very large vocabulary on-line cursive handwriting recognition. *on Frontiers in Handwriting Recognition*, pages 1–10, 1998.

[29] Joachim Schenk and Gerhard Rigoll. Novel hybrid NN/HMM modelling techniques for on-line handwriting recognition. In *in Proc. 10th Int. Workshop on Frontiers in Handwriting Recognition*, pages 619–623, 2006.

[30] M. Schuster and K.K. Paliwal. Bidirectional recurrent neural networks. *IEEE Transactions on Signal Processing*, 45(11):2673–2681, 1997.

[31] Andrew W Senior and Anthony J Robinson. An off-line cursive handwriting recognition system. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 20(3):309–321, 1998.

[32] T. Starner, J. Makhoul, R. Schwartz, and G. Chou. On-line cursive handwriting recognition using speech recognition methods. *IEEE Int.Conf. on Acoustics, Speech and Signal Processing*, 5:125–128, 1994.

[33] YH Tay and PM Lallican. An offline cursive handwritten word recognition system. *Electrical and Electronic Technology, 2001. TENCON. Proceedings of IEEE Region 10 International Conference on*, 2:519—-524, 2001.

[34] Poland Verma, B.K. ; Dept. of Comput. Sci., Warsaw Univ. of Technol. Handwritten Hindi character recognition using multilayer perceptron and radial basis function neural networks. In *Neural Networks*, volume 4, pages 2111 – 2115, 1995.

[35] C. Viard-Gaudin, P.M. Lallican, S. Knerr, and P. Binter. The IRESTE On/Off (IRONOFF) dual handwriting database. *Proceedings of the Fifth International Conference on Document Analysis and Recognition. ICDAR '99 (Cat. No.PR00318)*, pages 455–458, 1999.

[36] Alessandro Vinciarelli, Samy Bengio, and Horst Bunke. Offline Recognition of Unconstrained Handwritten Texts Using HMMs and Statistical Language Models. *iEEE Trans. on Pattern Analysis and Machine Intelligence*, 26(6):709–720, 2004.

[37] Juergen Vinciarelli, Alessandro and Luettin. offline cursive script recognition based on contineous density HMM. *Proc. 7th International Workshop on Frontiers in Handwriting Recognition, Amsterdam*, 2000.